Stream Reasoning using Temporal Logic and Predictive Probabilistic State Models

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Execution Monitoring in Robotics

Am I in a no-fly zone? - **boolean**
Is it **likely** that I am in a no-fly zone?
Is it **likely** that I am about to crash into the wall in the near future?
Execution Monitoring in Robotics

Am I in a no-fly zone? - **boolean**
Is it **likely** that I am in a no-fly zone?
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Metric Temporal Logic (MTL) formulas are evaluated over the stream (infinite state sequence) using Progression.

Incremental evaluation by formula re-writing to incorporate what has been observed so far.
What do we know (about terms) at $t_k$?
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- Observations
- Predictions (past)
- Estimates
- Predictions (future)
Truth values of predicates
Numerical values of terms
**Stochastic estimates of terms** (Green, solid outline)
**Stochastic predictions of terms** (Red, dashed outline)
An alternative view

\[ S_{t_0} \quad S_{t_1} \quad S_{t_2} \]
\[ S_{t_0|t_0} \quad S_{t_1|t_0} \quad S_{t_2|t_0} \]
\[ S_{t_0|t_1} \quad S_{t_1|t_1} \quad S_{t_2|t_1} \]
\[ S_{t_0|t_2} \quad S_{t_1|t_2} \quad S_{t_2|t_2} \]
P-MTL is MTL extended with a stochastic temporal term operator

Estimated feature: \( \bullet_{t|t} \text{Altitude}[\text{uav1}] (t \leq 0) \)

Predicted feature: \( \bullet_{t'|t} \text{Altitude}[\text{uav1}] (t \leq 0, \ t' \neq t) \)

\[ \square (\text{Altitude}[\text{uav1}] - \text{Altitude}[\text{roofA}]) > 2 \]

\( \square (Pr((\bullet_{0|0} \text{Altitude}[\text{uav1}] - \bullet_{0|0} \text{Altitude}[\text{roofA}]) > 2) \geq 0.99) \)

\( \square (Pr((\bullet_{3|0} \text{Altitude}[\text{uav1}] - \bullet_{0|0} \text{Altitude}[\text{roofA}]) > 2) \geq 0.99) \)
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Predicates

\[ P(\tau_1, \ldots, \tau_n) \mid \neg \alpha \mid \alpha \land \beta \mid \alpha \lor \beta \mid \alpha \rightarrow \beta \mid \Box_{t_1} \alpha \mid \Box_{[t_1, t_2]} \alpha \mid \Box \alpha \mid \Diamond_{[t_1, t_2]} \alpha \mid \Diamond \alpha \]

Terms

\[ \bar{f}[\text{const}] \mid \bullet_{t_1} \bar{f}[\text{const}] \mid \bullet_{t_1|t_2} \bar{f}[\text{const}] \mid \text{const} \mid f(\tau_1, \ldots, \tau_n) \mid \text{Pr}(g(\tau_p, c_1, \ldots, c_m)) \]
Grounding of P-MTL terms in computational environment

- Sensors
- Observations
- Probabilistic Reasoning
- Probabilities
- Logical Reasoner (Progression)
Grounding of P-MTL terms in computational environment

\[ \mathcal{E} = \langle T, O, F, \bar{F}, X, D, P \rangle \]

**Intuition**

P-MTL: Stochastic temporal term operator

**P-MTL: Syntax**

**P-MTL: Grounding**

Grounding of P-MTL terms in computational environment

- Sensors
- Observation: \( \bar{f}_t \)
- Temporal Model: \( \gamma_t^X \)
- Estimator
- Estimation: \( x_t | t \)
- Predictor
- Prediction: \( x_{t'} | t \)
- Logical Reasoner (Progression)
- Truth: \( b_t \)
- Probability: \( pr_t \)

**Summary**

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**Example: Execution Monitoring**

A UAV may only move under the conditions that

- **Its perception is precise**
  - The estimate of its position to be within a 1m radius circle with 99% probability

- **Its near-time predictions are precise**
  - The prediction of its position 3 seconds from now must be within a 1m radius circle with 95% probability

- **Its near-time prediction quality is high**
  - The prediction must match with the then estimated position with at least 50% similarity.
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• The prediction of its position 3 seconds from now must be within a 1m radius circle with 95% probability
• The prediction must match with the then estimated position with at least 50% similarity.

\[ \square \left( \Pr(insideRelative(\bullet_0|0\text{Position}[\text{uav1}], 1\text{mCircle})) > 0.99 \right) \wedge \]
\[ \Pr(insideRelative(\bullet_3|0\text{Position}[\text{uav1}], 1\text{mCircle})) > 0.95 \wedge \]
\[ \circ_3 \left( similarity(\bullet_0|0\text{Position}[\text{uav1}], \bullet_0|{-3}\text{Position}[\text{uav1}]) > 0.5 \right) \]
We introduce\(^1\) **P-MTL** as an extension to MTL

Our contribution is a formal interface between existing logical reasoning and existing probabilistic reasoning methods:

- A **formal framework** with an **explicit** separation
- A selection of important temporal and probabilistic concepts from probability theory can be referred to at the logical level
- Both aspects retain strengths and computational complexities